Five Year Integrated M.Sc. Examination, 2017

Semester - II

Course: MT-1-2-2 (Old)

(Scientific Computing)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.

Answer ***Question No.1 and any three*** questions from the rest.

1. a) In the decimal number 103522, the most significant digit is 1. The statement is (i) true (ii) false.
   b) The binary representation of $(93)_{10}$ is (i) 1011101 (ii) 1001101 (iii) 1101011 (iv) none.
   c) The decimal representation of $(0.123)_8$ is (i) 0.1234 (ii) (i) 0.4321 (i) 0.2344 (iv) none.
   d) 0.2 has (i) finite (ii) infinite binary representation.
   e) $\nabla + \Delta$ is equal with (i) $\nabla\Delta$ (ii) ∇ / Δ (iii) $(\nabla + \Delta)^{-1}$ (iv) none
   f) The diagonal dominance of a matrix $(a_{i,j})$ with $(i, j = 1,2,3,....,n)$ is defined as

   (i) $|a_{i,i}| \geq \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{i,j}|$ for all i (ii) $|a_{i,i}| > \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{i,j}|$ (iii) $|a_{i,i}| \leq \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{i,j}|$ for all i. 6x2=12

2. a) Use Gaussian elimination with back substitution to solve the following system

   $$x_1 - x_2 + x_3 = 2$$
   $$3x_1 + 2x_2 - 2x_3 = 1$$
   $$2x_1 - 2x_2 + 3x_3 = 6$$

   b) If $f(x) = e^{ax}$, show that $\Delta^n f(x) = (e^{ah} - 1)^n e^{ax}$.
   c) Find a polynomial $P(x)$ of degree at most 2 such that $P(0) = 1$; $P(1) = 1$ and $P(2) = 1$. 6+5+5=16

3. a) Write short notes on (i) Floating point representation (ii) Significant digit (iii) Errors.
   b) Derive composite Simpson's 1/3rd rule for numerical integration $\int_a^b f(x)dx$. State its geometrical significance.
   c) For the four interpolating nodes -1,1,3,4, what are the fundamental polynomials $L_i (0 \leq i \leq 3)$ required in the Lagrange interpolation procedure? (2.5+2.5)+(5+1)+5=16

4. a) Derive Newton backward difference interpolating polynomial by using $(n+1)$ equispaced data points.
   b) Compare the advantages and disadvantages of divided difference polynomials with Lagrange interpolating polynomial.
   c) Determine the least squares approximation of the type $ax^2 + bx + c$, to the function 2^x at the points $x_i = 0,1,2,3,4$. 5+5+6=16

5. a) Deduce Error formula in numerical integration.
   b) Perform five iterations of the Newton Raphson method to obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$.

P.T.O.

c) Prove that any polynomial of degree *n* may be written in the form

$$c_0 + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + c_3(x - x_0)(x - x_1)(x - x_2) + \ldots\ldots$$
$$+ c_n(x - x_0)(x - x_1)\ldots\ldots(x - x_{n-1})$$

where $x_0, x_1, x_2, \ldots. x_{n-1}$ are arbitrary points and $c_0, c_1, c_2, \ldots. c_{n-1}$ are constants. 5+6+5=16

6. a) Derive Hermite interpolating polynomial for $x_i, f_{i,}, f_i'$ where i=0,1,….n.

b) Determine the parameters in the formula $P(x) = a_0(x-a)^3 + a_1(x-a)^2 + a_2(x-a) + a_3$ such that $P(a) = f(a),\ P'(a) = f'(a),\ P(b) = f(b),\ P'(b) = f'(b)$.

c) If $f(x) = U(x)V(x)$, show that $f[x_0, x_1] = U[x_0]V[x_0, x_1] + U[x_0, x_1]V[x_1]$.

6+5+5=16