M.A./M.Sc. Examination 2017

Semester - IV

Mathematics

Course: MMC-41

(Differential Geometry)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer any four questions.

1. a) Define curvilinear coordinate system.
   Show that (x^1, x^2, x^3) defined by the following transformations

   $$y^1 = x^1 Sinx^2 Cosx^3$$
   $$y^2 = x^1 Sinx^2 Sinx^3$$
   $$y^3 = x^1 Cosx^2$$

   are curvilinear coordinates and also find the coordinate surfaces. 1+3+2

   b) Show that a necessary and sufficient condition for a given curvilinear coordinate system to be orthogonal is that $g_{ij} = 0$ for $i \neq j$ and $g_{ii} = 0$ for $i = 1, 2, 3$ at every point of the region of consideration in E^3. 4

2. a) Show that the intrinsic derivative of a tensor of type $(1,0)$ is again a tensor of type $(1,0)$. If A is a scalar then show that $\frac{\delta A}{\delta t} = \frac{dA}{dt}$. 2+1

   b) Prove that a curve with vanishing torsion and constant nonzero curvature is a circle. 3

   c) Prove that the magnitude of a parallel vector field along a curve is constant. Also show that the angle between two non-null parallel vector fields along a curve remains constant. 2+2

3. a) Prove that in order that the principal normals of a curve be binormals of another, the relation $a(\kappa^2 + \tau^2) = b\kappa$ must hold where a, b are constants. 4

   b) Show that $g_{ij} \frac{\delta x^i}{\delta s} \frac{\delta x^j}{\delta s} = \kappa^2 + \tau^2$ 2

   c) Show that the ratio of the curvature to the torsion of a space curve is a nonzero constant iff the curve is a helix. 4

4. a) A curve *C* is defined in cylindrical coordinates x^i as follows:
   $x^1 = a, x^2 = \theta, x^3 = b\theta \, (b \neq 0)$ where a, b are constants of which '*a*' is positive and θ is a function of s. Find the curvature and torsion of *C*. 5

   b) Prove that every plane curve is a Bertrand curve. 3

   c) Show that a typical example of Bertrand curve in E^3 is a circular helix. 2

P.T.O.

5. a) Show that the cosine of the angles θ between the coordinate curves on a surface S is given by

$$Cos\theta = \frac{a_{12}}{\sqrt{a_{11}}\sqrt{a_{22}}}.$$

Show that for the surface

$$x^1 = aCosu^1 \sin u^2,\ x^2 = aSinu^1 Sinu^2,\ x^3 = cCosu^2$$

where a, c are constants, the coordinate curves are orthogonal. 3+3

b) Prove that

$$\sigma^2 = a_{\alpha\beta}\frac{\delta t^\alpha}{\delta s}\frac{\delta t^\beta}{\delta s} \text{ and } \sigma = \varepsilon_{\alpha\beta} t^\alpha \frac{\delta t^\beta}{\delta s}$$ 2+2

6. a) Show that the geodesics in the Euclidean space E^3 with Cartesian coordinate system are straight lines. 2

b) Compute the geodesic curvature of the curve

$$C : u^1 = \text{Constant} = c \neq 0,\ u^2 = u^2$$

on the surface of the sphere

$$S : y^1 = a_1 Cosu^1 Cosu^2$$
$$y^2 = a_1 Cosu^1 Sinu^2$$
$$y^3 = a_1 Sinu^1$$

a_1 being a constant, u^1, u^2 are curvilinear coordinates on the surface of the sphere, y^i, $i = 1, 2, 3$ are orthogonal Cartesian coordinates. 4

c) Show that the Gaussian curvature is invariant. 4