M.A./M.Sc. Examination, 2017

Semester - II

Mathematics

Paper: MMC-26

(Numerical Analysis)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions

1. a) Obtain the cubic spline interpolation for the function $f(x)$ whose tabular values are known at $(n+1)$ distinct equispaced points by using piecewise cubic Hermite interpolation formula under natural spline conditions. 6

   b) Define $f(x_0, x_0, ..., x_0)$ for coincident values of $(n+1)$ arguments and hence deduce Taylor's formula with Lagrange's form of remainder. 4

2. a) Obtain the best uniform polynomial approximation $P_n(x)$ of degree n in the minimax sense to the function $f(x)$ which is continuous on $[a, b]$. 6

   b) Use Lanczos economization to approximate $f(x)=(2x-1)^3$ by a straight line on the interval [0, 1], so that the maximum norm of the error function is minimized. 4

3. a) Explain briefly the Romberg's quadrature rule for integration of $f(x)$ over the interval $[a, b]$. Point out the stopping criterion based on either a row or a column convergence test. Comment also on the accuracy of the method. 6

   b) Determine the coefficients in the formula

   $$\int_0^{2h} x^{-1/2} f(x)\,dx = (2h)^{1/2}\left[a_0 f(0) + a_1 f(h) + a_2 f(2h)\right] + R$$

   and calculate the remainder R, when $f'''(x)$ is constant. 4

4. a) Explain briefly the power method for the largest eigen pair of a real numerical matrix including its stopping criterion. State how the least eigen pair can be obtained for both the cases of singular and non-singular matrices and also how other remaining eigen pairs are obtained. 6

   b) Transform the matrix $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ to tridiagonal form by Givens' method. 4

5. a) Briefly describe Adams' method for solving numerically the well-posed initial value problem $\frac{dy}{dx} = f(x, y),\ y(a) = y_0$ in a finite interval $[a, b]$. 6

   b) Prepare a scheme corresponding to the single step method for the differential equation $\frac{dy}{dx} = f(x, y)$ which produces exact result for $y(x) = a + be^{-x}$; a, b being constants. 4

P.T.O.

6. a) Briefly describe an explicit finite difference scheme for solving the problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \; c > 0, \; t > 0, \; 0 < x < l$$

with $u(0, t) = 0 = u(l, t), \; t > 0$

$$u(x, 0) = F(x) \text{ and } \frac{\partial u}{\partial t}(x, 0) = G(x), \; 0 \le x \le l$$

in which $F(x)$ and $G(x)$ are known functions of x. 6

b) Discuss the shooting method for solving boundary value problem

$$-u'' + p(x)u' + q(x)u = r(x), \; a < x < b$$

subject to the given conditions $u'(a) = \gamma_1, \; u'(b) = \gamma_2$ where $p(x), \; q(x)$ and $r(x)$ are assumed to be continuous on $[a, b]$ and γ_1, γ_2 are non-zero real constants. 4

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