M.A./M.Sc. Examination, 2017

Semester - II

Mathematics

Paper: MMC-25

(Solid Mechanics)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

(Notations and symbols have their usual meaning unless otherwise stated)

Answer ***any four*** questions

1. a) Prove that the extremum values of normal strains at a point of a continuum are principal strains. 5

   b) The displacement components in an elastic solid are given as follows:

   $u_1 = \epsilon \left(X_1 + 2X_2 + 3X_3\right)$

   $u_2 = \epsilon \left(-2X_1 + X_2\right)$

   $u_3 = \epsilon \left(X_1 + 4X_2 + 2X_3\right)$

   where ϵ is a small quantity. Calculate dilatation, rotation and principal strains. 5

2. a) Write down the expressions for Lagrangian and Eulerian finite strain tensor. Show that, in case of infinitesimal deformation, the distinction between them disappears. 1+4

   b) For the deformation $u_i = A_{ij}X_j$, where A_{ij} are constants $\left(i, j = 1, 2, 3\right)$, determine an expression for change of volume per unit original volume. If A_{ij} are very small, show that it reduces to cubical dilatation. 5

3. a) Prove that the stress vector at a point on any arbitrary plane surface is a linear function of three stress vectors acting on any three mutually perpendicular planes through that point. 5

   b) The stress tensor at a point is given by

   $\left(\tau_{ij}\right) = \begin{pmatrix} 0 & 1 & 2 \\ 1 & b & 1 \\ 2 & 1 & 0 \end{pmatrix}$ where b is a constant.

   Determine b so that stress vector on some plane at the point will be zero. Determine the direction cosines of the normal to that plane. 5

4. a) State the principles of conservation of mass in both material and spatial methods. Write down their equations. Show that these two forms are equivalent. 2+1+3

   b) Given the following stress distribution

   $\left(\tau_{ij}\right) = \begin{pmatrix} x_2 & -x_3 & 0 \\ -x_3 & 0 & -x_2 \\ 0 & -x_2 & T \end{pmatrix}$, find T such that

   the stress distribution is in equilibrium with body force $\vec{F} = -g\hat{e}_3$. 4

5. a) Define the first BVP of elastostatics and obtain the compatibility equation for stresses. 1+4

   b) In case of equilibrium of elastic body under zero body forces, prove that the first strain invariant Δ and rotation tensor ω_{ij} are harmonic functions. 5

P.T.O.

6. a) Derive the displacement components in plane elasticity from Airy's stress function $\chi(x, y)$. 5

   b) When a body is subjected to uniform pressure such that

   $\tau_{11} = \tau_{22} = \tau_{33} = -p,\ \tau_{23} = \tau_{31} = \tau_{12} = 0.$

   Prove that $\frac{u_1}{x_1} = \frac{u_2}{x_2} = \frac{u_3}{x_3} = \frac{-p}{3K}$, assuming that the displacement and rotation at $x_1 = 0,\ x_2 = 0,\ x_3 = 0$ are zero. 5

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