M.A./M.Sc. Examination, 2017
Semester - II
Mathematics
Course: MMC-22
(Topology)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Answer ***any four*** questions

1. a) State and prove Schroeder-Berstein theorem. 5
   b) Show for any set A, $\overline{\overline{A}} < \overline{\overline{\wp(A)}}$. 3
   c) Prove that any nondegenerated interval has the same cardinality as that of $\mathbb{R}$. 2

2. a) If τ_l and τ_u be respectively the lower limit and upper limit topologies on $\mathbb{R}$ then show that
      i) if τ is a topology on $\mathbb{R}$ containing $\tau_l \bigcup \tau_u$ then τ is the discrete topology on $\mathbb{R}$. 2
      ii) If $\tau = \tau_l \bigcap \tau_u$ then τ is the usual topology on $\mathbb{R}$. 2
   b) Let X be a nonempty set and for each $x \in X$, η_x be a family of subsets of X satisfying
      i) $U \in \eta_x \Rightarrow x \in U$
      ii) $U, V \in \eta_x \Rightarrow U \bigcap V \in \eta_x$
      iii) $U \supset V \in \eta_x \Rightarrow U \in \eta_x$
      iv) For $U \in \eta_x$ there exists $V \in \eta_x$ such that $V \subset U$ and $V \in \eta_y \forall y \in V$.

      Then show that there is a unique topology τ on X such that η_x coincides with the family of all τ- neighbourhoods of x. 4
   c) Show that in a topological space X, $X|\overline{A} = (X|A)^{\mathrm{o}}$, $A \subset X$. 2

3. a) Prove that a set A in a topological space X is dense iff A intersects every nonempty open set in X. Is the set of reals with cocountable topology separable? Answer with justifications. 3+2
   b) Prove that every 2nd countable space is Lindeloff. Give an example of a 1st countable space which is not 2nd countable. 3+2

4. a) Prove that a topological space is T_1 iff every single pointic set is closed. Give an example of T_1 space which is not T_2. 2+2
   b) Prove that every completely regular space is a regular space. 3
   c) Prove that a topological space is normal iff for any closed set F and for any open set $U \supset F$ there is an open set V such that $U \supset \overline{V} \supset V \supset F$. 3

5. a) Prove that every compact Hausdorff space is normal. 4
   b) Prove that every real-valued continuous function over a compact space is bounded and attains its bounds. Give an example to show that the result does not hold if the compactness of the space is withdrawn. 4+2

6. a) Prove that a set of real numbers is connected with respect to usual topology iff it is an interval. Prove that continuous image of a connected space is continuous. Prove that no function over $\mathbb{R}$ with rational values is continuous with respect to usual topology. 3+2+1
   b) Prove that cartesian product of two connected spaces is connected. 4
