M.A./M.Sc. Examination 2017

Semester - IV

Mathematics

Optional Course: MMO-41 (P4)

(Advanced Complex Analysis-II)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
*Answer **any four** questions.*

1. a) Suppose that f is analytic in $|z| < R$ and $f(0) \neq 0$. Let $r_1, r_2, \ldots$ be the moduli of zeros of f in $|z| < R$, arranged in the non-decreasing order. If $r_n \leq r < r_{n+1}$, then show that $\log \frac{r^n |f(0)|}{r_1 . r_2 \ldots r_n} = \frac{1}{2\pi} \int_0^{2\pi} \log |f(re^{i\theta})| d\theta,$ where a zero of order p is counted p times. 3

   b) Let $f(z)$ be regular and $|f(z)| \leq M$ in $|z| \leq R$. Suppose that $f(0) \neq 0$. Then show that the number of zeros of $f(z)$ in $|z| \leq \delta R$, where $0 < \delta < 1$, does not exceed $\frac{1}{\log \frac{1}{\delta}} \log \frac{M}{|f(0)|}$. 2

   c) State and Prove Poisson-Jensen formula. 5

2. a) Define the Nevanlinna's characteristic function $T(r, f)$. Find $T(r, f)$ when $f(z) = e^{3z}$. 1+2

   b) If $f_1, f_2, \ldots, f_p$ be p meromorphic functions, then show that $T\left(r, \sum_{\nu=1}^{p} f_\nu(z)\right) \leq \sum_{\nu=1}^{p} T(r, f_\nu(z)) + \log p.$ 2

   c) State and prove Nevanlinna's first fundamental theorem. 3

   d) Show that if $0 < |a| < 1$, $|T(r, f) - T(r, f + a)| \leq \log 2$. 2

3. a) For any 'a', show that $N(r, a)$ is an increasing convex function of $\log r$. 2

   b) Prove that $\frac{1}{2\pi} \int_0^{2\pi} m(r, e^{i\theta}) d\theta \leq \log 2.$ 3

   c) Suppose that $S(r)$ is a nonnegative and nondecreasing function defined for $r \geq r_0 > 0$. If $S(r)$ is of order $\rho \left(0 < \rho < \infty\right)$, then prove that $\int_{r_0}^{\infty} \frac{S(r)}{r^{k+1}} dr$ converges or diverges according as $k > \rho$ or $k < \rho$. 4

   d) State Nevanlinna's second fundamental theorem. 1

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4. a) If $f(z)$ is regular in $|z| \le R,$ then show that

$$T(r,f) \le \log^{+} M(r,f) \le \frac{R+r}{R-r} T(R,f), \quad 0 \le r < R.$$ 3

b) Find the order of $f(z) = \dfrac{z-1}{(z+1)^2(z+2)}$. 2

c) If ρ_1, ρ_2 be the orders of the meromorphic functions $f_1(z)$ and $f_2(z)$ respectively, then show that order of $f_1.f_2 \le \max\{\rho_1, \rho_2\}$. 3

d) If $f(z)$ be a meromorphic function with $f(0) \ne 0, \infty$, then show that $\rho(f) = \rho\left(\frac{1}{f}\right)$, where $\rho(f)$ is the order of f. 2

5. a) Suppose that $f(z)$ is a non-constant meromorphic function in the complex plane and $a_1, a_2, ..., a_q (q \ge 3)$ are q distinct values in the extended complex plane then prove that $(q-2)T(r,f) < \sum_{j=1}^{q} \overline{N}\left(r, \frac{1}{f-a_j}\right) + S(r,f)$, where $S(r,f)$ is defined by second fundamental theorem. 3

b) Define $\delta(a,f)$, $\theta(a,f)$ and $\Theta(a,f)$ and find the relation among them. 4

c) If $f(z) = a$ has only multiple roots, show that $\Theta(a) \ge \frac{1}{2}$. 2

d) State Nevanlinna's theorem on deficient functions. 1

6. a) Let f and g be nonconstant rational functions. If f and g share distinct values a, b CM, then show that there exists a non-zero constant k satisfying

$$\frac{f-a}{f-b} = k\frac{g-a}{g-b}.$$

Further show that if there exists a point α such that $f(\alpha) = g(\alpha) \notin \{a,b\}$ then $f = g$. 4

b) State and Prove Milloux's theorem. 6

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