

M.A./M.Sc. Examination 2017
Semester - IV
Mathematics
Course: MMO-41 (P2) (Optional)
(Advanced Topology-II)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.
 Notations and symbols have their usual meanings.

Answer Question No. 6 and any three from the rest.

1. a) Prove that every regular space having countable locally finite base is normal. 5
 b) State and prove Nagata-Smirnov metrization theorem. 7

2. a) Let (X, U) be a uniform space and D be the family of all pseudometrics on X which are uniformly continuous from $(X \times X, U \times U)$ to $(\mathbb{R}, U_{\mathbb{R}})$ ($U_{\mathbb{R}}$ is the usual uniformity on \mathbb{R}). Prove that D generates U . 4
 b) Let U be the uniformity generated by a family D of pseudometrics on a set X . For each $d \in D$, let U_d be the uniformity induced by the pseudometric d and $Y = \prod_{d \in D} (X_d, U_d)$, $X_d = X$, $\forall d \in D$. Prove that the evaluation mapping $f : X \rightarrow Y$ defined by $(f(x))(d) = x$, $\forall d \in D$, $x \in X$, is a uniform embedding. 4
 c) Prove that a topological space is uniformizable iff it is completely regular. 4

3. a) Prove that a convergent net in a uniform space is Cauchy. Is the converse true? Answer with reasons. Prove that a Cauchy net is convergent iff it has a cluster point. 5
 b) Prove that a uniform space is compact iff it is complete and totally bounded. 4
 c) Prove that every continuous function from a compact uniform space to a uniform space is uniformly continuous. 3

4. a) Give an example of a topological group. 3
 b) If H is an algebraic subgroup of a topological group G then show that \bar{H} is also a subgroup of G . Also show that every open subgroup is a clopen subgroup. 3
 c) Prove that in a topological group every component containing the identity element is a closed normal subgroup. 3
 d) For any open nbd U of e of a connected topological group G , show that $G = \bigcup_{n \geq 1} U^n$ 3

5. a) Let (G, τ) be a topological group and H be a normal subgroup of G . Then prove that $(G/H, \tau')$ is a topological group where τ' is the quotient topology induced by the canonical mapping $\phi: G \rightarrow G/H, \phi(x) = xH, x \in G$. Also show that ϕ is an open homomorphism from (G, τ) to $(G/H, \tau')$. 4
- b) Let α be an algebraic homomorphism from a topological group (G_1, τ_1) into a topological group (G_2, τ_2) and let H be the kernel of α . Further let $\alpha = \alpha_0 \circ \phi$ for some $\alpha_0: G/H \rightarrow G_2$, where $\phi: G_1 \rightarrow G/H$ is the canonical mapping. Prove that $\alpha: (G_1, \tau_1) \rightarrow (G_2, \tau_2)$ is continuous or open iff $\alpha_0: (G/H, \tau') \rightarrow (G_2, \tau_2)$ is continuous or open (τ' is the quotient topology on G/H). Hence show that if $\alpha: (G_1, \tau_1) \rightarrow (G_2, \tau_2)$ is open homomorphism then the topological group $(G/H, \tau')$ is isomorphic to (G_2, τ_2) . 4
- c) Prove that every locally compact Hausdorff topological group is normal. 4
6. Answer any two : 2×2
- (i) Give an example of a 2nd countable space which is not metrizable.
 - (ii) Show that if a topological group is T_0 then it is T_3 .
 - (iii) Prove that a topological group is homogeneous.
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