M.A./M.Sc. Examination 2017

Semester - IV

Mathematics

Course: MMO-41 (Optional)(A10)

(Magneto-hydrodynamics and Magnetized Plasmas-II)

Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer any four questions.

1. a) Discuss the propagation of EM waves in a cold electron (ions form the background) plasma in presence of a uniform magnetostatic field. Show that the dispersion relation can be obtained in the form

$$\left(S\sin^2\theta + P\cos^2\theta\right)\eta^4 - \left[RL\sin^2\theta + SP\left(1+\cos^2\theta\right)\right]\eta^2 + PRL = 0,$$

where θ is the angle of the wave vector with the magnetic field $\vec{B} \equiv B_0\hat{z}$, η is the index of refraction and other symbols have their usual meanings with $R = S + D,\ L = S - D,\ S = \frac{1}{2}\left(R + L\right),\ D = \frac{1}{2}\left(R - L\right)$.

Hence obtain the Appleton – Hartree equation. 6+4

2. a) The electric field perturbation of an EM wave propagating along the constant magnetic field $B_0\hat{z}$ is given by

$$\begin{pmatrix} S-\eta^2 & -iD & 0 \\ iD & S-\eta^2 & 0 \\ 0 & 0 & P \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0,$$

where $S = 1 - \frac{X}{1-Y^2},\ D = -\frac{XY}{1-Y^2},\ P = 1 - X$ with $Y = \omega_{cc}/\omega,\ X = {\omega_{pc}}^2/\omega^2$.

Obtain the expressions for η for RCP and LCP wave modes. Discuss the polarization of these two modes. 3+2

b) Define ordinary (O) and extraordinary (X) waves for the propagation of EM waves perpendicular to the constant magnetic field $B_0\hat{z}$. Hence obtain the reflection and resonance points for these waves. 3+(1+1)

3. Write short notes on (i) Atmospheric whistlers (ii) Faraday rotation. 5+5

4. Consider the propagation of waves in a fully ionized warm plasma composed of electrons and singly charged positive ions with no external magnetic field. Show that the dispersion relation for longitudinal waves can be written as

$\left(\omega^2 - {\omega_{pe}}^2 - k^2{C_{se}}^2\right)\left(\omega^2 - {\omega_{pi}}^2 - k^2{C_{si}}^2\right) - {\omega_{pe}}^2{\omega_{pi}}^2 = 0$.

Hence obtain the Bohm-Gross dispersion relation. Discuss the cases of high-frequency and low-frequency oscillations. 5+2+1 $\frac{1}{2}$+1 $\frac{1}{2}$

P.T.O.

5. Explain what is meant by the term pinch effect in the confinement of a plasma by a magnetic field. Discuss the phenomena of Bennett pinch. 3+7

6. What is plasma sheath? Estimate the electric potential on the wall after the formation of the plasma sheath. 2+8

———