M.A./M.Sc. Examination, 2017
Semester-IV
Mathematics
Course: MMO-41 (A9)
(Lie Group of Transformations and Partial Differential Equations)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Symbols have their usual meaning.

Answer ***Question No. 1*** and any ***three*** from the rest.

1. Distinguish Lie group of point transformations (LGPT) admitted by a differential equation and continuous group of equivalence transformations admitted by a family of differential equations.

   Derive expression for $\phi^{(i)}, \mu f^{(k)}, \mu g^{(k)}$ in the prolongation of infinitesimal generator (IG) $\hat{X} = \xi \partial_x + \tau \partial_t + \phi \partial_u + \mu f \partial_f + \mu g \partial_g$ for the continuous group of equivalence transformations admitted by the family of equation $f(u)u_t = g(u_x)u_{xx}$. 3+10

   OR

   Derive determining equations and find explicit forms of ξ, τ, f, g containing atleast two arbitrary constants involved in the IG $\hat{X} = \xi \partial_x + \tau \partial_t + (f u + g)\partial_u$ by Lie group of symmetry transformations (LGST) admitted by the wave equation $u_{tt} = u_{xx}$. It may be assumed that ξ, τ, f, g are multinomial in *x* and *t* of order atmost 2. 6+7

2. Verify whether the generalized KdV equation $u_t + u^{\nu} u_x + u_{xxx} = 0$ admits one parameter LGST generated by the IG $\hat{X} = x\partial_x + 3t\partial_t - \frac{2}{\nu} u\partial_u$. Reduce the partial differential equation mentioned above to an ordinary differential equation with the help of similarity transformations corresponding to the given IG $\hat{X}$, if yes. 9

3. Establish the recursion relation

   $$\phi^{(k)}_{i_1, i_2, ..., i_{k-1} i_k} = D_{i_k}\left[\phi^{(k-1)}_{i_1 ... i_{k-1}}\right] - D_{i_k}\left[\xi_j\right] u_{i_1, i_2, ..., i_{k-1} j}$$

   involved in the prolongation of IG for LGPT defined on geometric space whose points are described by $(x_1, ..., x_n, u)$ where the variable *u* may be a function of other variables $x_1, ..., x_n$. Here $i_1, ..., i_k = 1, 2, ..., n, k \geq 1$.

   Provide the statement in MATHEMATICA for getting prolongation of a given IG $\hat{X} = \xi \partial_x + \tau \partial_t + \phi \partial_u$. 8+1

4. What do you mean by similarity variables associated with a LGST generated by the IG $\hat{X} = \xi \partial_x + \eta \partial_y + \phi \partial_u$ admitted by a differential equation

   $$F\left(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}\right) = 0?$$

   Find similarity variables for $\hat{X} = y\partial_x + \left(x + y^2\right)\partial_u$. 3+6

P.T.O.

5. Find the group invariant solution of heat equation $u_t = u_{xx}$ corresponding to the IG

$$\hat{X} = xt\partial_x + t^2\partial_t - \left(\frac{x^2}{4} + \frac{t}{2}\right)u\partial_u.$$

Provide statement for obtaining similarity variables for $\hat{X}$ mentioned above with the help of MATHEMATICA. 8+1

6. Show that the heat conduction equation $u_t = u_{xx}, (x, t) \in \mathbb{R}^2$ admits six parameter Lie group of point transformations generated by $\hat{X} = \xi\partial_x + \tau\partial_t + \phi\partial_u$, where $\xi = \kappa + \beta x + \delta t + \gamma xt$, $\tau = \alpha + 2\beta t + \gamma t^2$, $\phi = \left\{\lambda - \frac{1}{2}\delta x - \gamma\left(\frac{x^2}{4} + \frac{t}{2}\right)\right\}u$. Find the values of six parameters $\alpha, \beta, \gamma, \delta, \lambda$ whenever the domain is restricted to $x \geq 0, t \geq 0$. 6+3

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