M.A./M.Sc. Examination, 2017

Semester - IV

Mathematics

Paper: MMO-41 (A2)

(Dynamical Oceanography)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Symbols having their usual meanings

Answer ***any four*** questions

1. a) Obtain Gibb's-Duhem relation for a multicomponent thermodynamic system. Deduce the corresponding relation for sea-water, supposing it to be a mixture of salt and pure water. 4+2

   b) Derive the following relations in (T, p, s) variables:

   i) $\Gamma = \dfrac{T}{C_p}\dfrac{\partial \tau}{\partial T}$

   ii) $d\eta = \dfrac{C_p}{T}dT - \dfrac{d\tau}{dT}dp - \dfrac{d\mu}{dT}ds.$ 2+2

2. a) Assuming the sea-water to be a two-component mixture of salt and pure water, show that the principle of conservation of mass yields the following equations:

   $$\frac{D\rho}{Dt} + \rho\nabla\cdot\vec{q} = 0 \text{ and } \rho\frac{Ds}{Dt} = -\nabla\cdot\vec{I}_s.$$

   Explain the significance of the quantities $\vec{I}_s$ and $\vec{I}_w$ in this connection. 3+2+1

   b) Obtain the boundary conditions at the free ocean surface by taking into account that the mass exchange process across the surface amounts to a flux 'b' of pure water in unit time per unit area. 4

3. State the assumptions on which the Boussinesq approximation of the governing equations of sea water is derived. Derive the approximate forms of the equation of continuity, equation of motion and equation of energy. 10

4. Assuming sea-water to be a viscous compressible heat-conducting fluid, derive the equation of conservation of energy in the form

   $$\frac{\partial}{\partial t}(\rho E_m) = -\nabla\cdot\vec{I}_E.$$

   Explain the significance of different components of E_m and $\vec{I}_E$. 10

5. a) When is sea-water said to be vertically stratified? Show that a vertically stratified fluid is in stable mechanical equilibrium if

   $$\frac{g}{\rho}\frac{dp}{dz} + \frac{g^2}{c^2} < 0.$$ 4

   b) Obtain the equation of conservation of energy for linearized small amplitude wave motion of ocean water. 6

6. Show that the problem of determination of frequencies of free oscillations in a horizontally unbounded ocean of constant depth may be reduced to the problem of solution of independent eigen value problems.

   Assuming sea-water to be incompressible and having a uniform Väisälä frequency, find approximately the eigen values of problem *V*. 10

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