B.A. (Honours) Examination, 2016

Semester-III

Integrated Mathematics and Statistics

Course – S-1.3.5.P.5 (Subsidiary)

(Calculus-I)

Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions.

1. (a) Evaluate the following limits

(i) $\lim_{x\to 1}\frac{x-1}{\sqrt{2x-1}-1}$ (ii) $\lim_{x\to\infty}\frac{x^2+x}{x+1}$

(b) For each of the following functions, indicate the value of x for which the function is discontinuous

(i) $h(x)=\frac{x+1}{x^2-1}$ (ii) $f(x)=\frac{x-1}{x^2+2x-3}$ 3+3+2+2

2. (a) Differentiate directly (from first principles) $f(x)=\log x$.

(b) Calculate the derivates

(i) $g(x)=\frac{e^{x^2}}{(2x-1)^3}$ (ii) Solve for y : $\ln(y+1)+\ln(y-1)=2x+\ln x$ 4+3+3

3. (a) Consider the maximisation problem: 5+5

max $f(x)=5x-x^2$

(i) Find the maximum value of the function for the unconstrained problem.

(ii) Confirm that the second order condition is satisfied.

(iii)What is the maximum value of the function if $x\le 2$

(b) For the function $f(x)=x^2$ find the Taylor's formula for n=2. (upto the second derivative). Show that using $dy=f'(x)\,dx$ to estimate the impact on y of a change in x of amount dx-leads to an underestimate of the actual change in y.

4. Consider the multivariate function 2+3+5

$f(x_1,x_2)=\left[0.3x_1^{-2}+0.7x_2^{-2}\right]^{-1/2}$

(a) Is this function homogeneous? If so what is the degree of homogeneity.

(b) Is this function concave or convex?

(c) Maximise $f(x_1,x_2)$ subject to the linear constant

$g(x_1,x_2)=ax_1+bx_2\le c$

Find the first and second order conditions.

P.T.O.

5. (a) Consider the pair of simultaneous equations (linear) 2+2+6

$$I+G=S$$

$$\bar{M}=M^d$$

where $I=10+0.1Y-3r$

$$S=-15+0.2Y+r$$

$$\bar{G}=50$$

$$\bar{M}=100$$

$$M^d=30-2r$$

Where Y, r are endogenous valuables while $\bar{M}$ and $\bar{G}$ are parameters

(i) Find the Equilibrium values of (Y,r)

(ii) How do these values change if $\bar{M}$ increases to 110 and G to 55

(b) Suppose the equations are now

$I(Y,r)+G=S(Y,r)$, $I_y>0, I_r<0$

$S_y>0, S_r>0$

$\bar{M}=K(Y,r)$ $K_y>0, K_r<0$

Find the signs of $\frac{dY}{dG}, \frac{dY}{dM} \frac{dr}{dG}$ and $\frac{dr}{dM}$.

6. (a) Evaluate the following integrals 2+2+3+3

(i) $\int 6xe^{x^2}dx$ (ii) $\int \frac{2x^3+1}{x^4+3x}$

(b) Evaluate the following integrals. Use the information provided to determine the constant of integration

(iii) $F(x)=\int(5x^3+2x+6)dx$, $F(0)=0$ (iv) $F(x)=\int x^{-1/2}.dx$, $F(4)=8$

7. (a) Evaluate the definite integral 2.5+2.5+5

(i) $\int_0^8 x^{2/3}dx$ (ii) $\int_{-1}^{1} 2e^x dx$

(b) Directly estimate $\int_0^b x^2$ as the area under the curve $f(x)=x^2$ between 0 and b.

8. (a) Integrate by parts 3+3+2+2

(i) $\int \frac{x}{\sqrt{1+x}}$ (ii) $\int(\ln x)^2 dx$

(b) Integrate by substitution

(i) $\int(4e^x+2x^2)(e^x+x)dx$ (ii) $\int \frac{2x}{(x^2+2)^{10}}dx$
