B.A. (Honours) Examination, 2016

Semester-III

Integrated Mathematics and Statistics

Course – S-1.3.5.P.4 (Subsidiary)

(Algebra-II)

Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions.

1. (a) Define vector addition and scalar multiplication with diagrams.
   (b) Define unit vector. Find the vector of length five which points in the opposite direction of (–1,2,–3).
   (c) Prove that $\|rv\| = |r|\,\|v\|$ for all r in R^1 and v in R^n. 4+4+2=10
2. (a) Write the vector v = (1,–2,5) as a linear combination of the vectors u_1=(1,1,1), u_2=(1,2,3), u_3=(2,–1,1)
   (b) Find k so that u = (2,3k,–4,1,5) and v = (6,–1,3,7,2k) are orthogonal.
   (c) Find a parametric representation of the line L in R^4 passing through P (4,–2,3,1) in the direction of u=(2,5,–7,8). 4+3+3=10
3. (a) Let u = 2i–3j+4k, v = 3i+j–2k, w=i+5j+3k. Find $u.v$ and $u.w$.
   (b) Derive parametric and non parametric equations for the plane through (6,0,0), (0,–6,0), (0,0,3)
   (c) Define linear independence. 4+4+2=10
4. (a) Let $A=\begin{bmatrix} 1 & 2 & 3 \\ -4 & -4 & -4 \\ 5 & 6 & 7 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & -5 & 1 \\ 0 & 3 & -2 \\ 1 & 2 & -4 \end{bmatrix}$
   Show that tr (AB) = tr (BA).
   (b) Find the inverse of $A=\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ 5+5=10
5. (a) Suppose $A=\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ and $(x) = x^2 + 3x - 10$ then show that A is a zero or root of g (x).
   (b) Find a 2×2 orthogonal matrix P whose first row is a multiple of (3,–4).
   (c) Suppose A is invertible show that if AB=AC then B=C. 4+4+2=10
6. (a) Prove without expanding

$$\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$$

   (b) Solve by Cramer's rule :

$$2x - z = 1,\ 2x + 4y - z = 1,\ x - 8y - 3z = -2$$ 5+5=10

7. (a) Find the adjoint and the reciprocal determinant of

$$\begin{vmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix}$$

(b) Find the rank of the matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$ 5+5=10

8. (a) Find the characteristic polynomial of $A=\begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & 3 & 9 \end{bmatrix}$

(b) For a given matrix $A=\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$. Find the eigenvalues and eigenvectors. 4+6=10
