

Five Year Integrated M.Sc. Examination, 2017

Semester - VIII

Course: PH-4-8-2A

(Advanced Quantum Mechanics-III)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin

Answer **Question No. 1** and any **three** from the rest

1. Answer any **five** from the following:

5 × 2 = 10

- How do you interpret the negative energy solutions of Dirac equation?
- "Schrodinger equation is not a relativistically covariant equation" - Explain.
- Find the solutions of Dirac equation in rest frame.
- Write down the infinitesimal Lorentz transformation matrix for a Lorentz boost along positive z -direction .
- Prove that $Tr\{\gamma^\mu\gamma^\nu\} = 4g^{\mu\nu}$.
- What do you mean by scattering state? How dose it differ from bound state?
- Define differential cross section. For what type of potential it would be independent of azimuthal angle ϕ ?

2. (a) Starting from the Dirac Hamiltonian in standard form i.e. $H = \vec{\alpha}\cdot\vec{p} + \beta m$, obtain the anticommutation relations among $\alpha^1, \alpha^2, \alpha^3$ and β . Now define γ matrices and show that $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}$.

(b) Derive continuity equation for the solutions of Dirac equation. Can you interpret these solutions as wave function in usual sense? Discuss briefly. (4+2)+(3+1)=10

3. (a) Suppose, ψ is a solution of Dirac equation and it transforms as

$$\psi(x) \rightarrow \psi'(x') = S(\Lambda)\psi(x),$$

under Lorentz transformation (LT). Here, Λ is the Lorentz transformation matrix. Find out the constraint on $S(\Lambda)$, for which Dirac equation remains form invariant under LT.

(b) Suppose for infinitesimal LT, that S matrix can be expressed as $S = \mathbb{1} + \tau$, where τ is infinitesimal matrix. Now, using appropriate representation of infinitesimal LT, prove that

$$\gamma^\mu\tau - \tau\gamma^\mu = \Delta\omega_\nu^\mu\gamma^\nu. \quad 5+5=10$$

4. (a) Prove the following identity :

$$(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{b}) = \vec{a} \cdot \vec{b} + i\vec{\sigma} \cdot (\vec{a} \times \vec{b}),$$

where, σ^i is the i -th component of Pauli matrices which satisfy the following relation :
 $\sigma^i \sigma^j = \delta_{ij} + i\epsilon^{ijk} \sigma^k$.

- (b) Write down the Dirac equation for a charged fermion moving in electromagnetic field.
 (c) Show that, in non relativistic limit, the above equation reduces to the following form

$$i\hbar \frac{\partial \phi}{\partial t} = \left[\frac{1}{2m} \left(\vec{p} - \frac{e}{c} \vec{A} \right)^2 - \frac{e\hbar}{2mc} \vec{\sigma} \cdot \vec{B} + e\Phi \right] \phi,$$

where ϕ is two component Pauli spinor.

3+1+6=10

5. (a) Given an infinitesimal Lorentz transformation

$$\Lambda_{\mu}^{\nu} = \delta_{\mu}^{\nu} + \omega_{\mu}^{\nu},$$

show that the infinitesimal parameters $\omega_{\mu\nu}$ are antisymmetric.

- (b) Suppose S_P is the operator which controls the transformation property of the Dirac spinor under Parity. Assuming Parity symmetry of Dirac theory show that the Parity operator is given by

$$S_P = e^{i\phi} \gamma^0 P^{(0)},$$

where, $P^{(0)}$ causes the spatial reflection $\vec{r} \rightarrow -\vec{r}$.

- (c) Show that the intrinsic parity of particles and antiparticles are opposite. 4+4+2=10

6. (a) In partial wave analysis, the scattering amplitude is given by

$$f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} (-i)^{l+1} \sqrt{\frac{2l+1}{4\pi}} C_l P_l(\cos \theta),$$

where all notations have their usual meaning. Moreover, we know from Rayleigh's formula that e^{ikz} can be expressed as follows

$$e^{ikz} = \sum_{l=0}^{\infty} (i)^l (2l+1) j_l(kr) P_l(\cos \theta).$$

Hence calculate the partial wave scattering amplitude (C_l) and total scattering cross-section for Hard sphere scattering.

- (b) By direct substitution, check that

$$G(\mathbf{r}) = -\frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{4\pi r},$$

satisfies Helmholtz equation $(\nabla^2 + k^2)G(\mathbf{r}) = \delta^3(\mathbf{r})$. (Use $\nabla^2 \left(\frac{1}{r}\right) = -4\pi\delta^3(\mathbf{r})$) 6+4=10