# B. Sc. (Honours) Semester-VI Examination 2017
# Statistics (Honours)
# Course: BSC-62
# Linear Models

**Time : Three Hours** **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. (a) Consider the following linear model

$$\underline{Y}^{n\times 1} = X^{n\times p}\underline{\beta}^{p\times 1} + \underline{\in}^{n\times 1}$$

With $E(\underline{\in}) = \underline{0}$, and $Var(\underline{\in}) = \sigma^2 I.$

Show that $\sigma^2$ can be unfiasedly estimated by

$$\hat{\sigma}^2 = \frac{1}{n-p}(\underline{y} - X\hat{\beta}_{mle})'(\underline{y} - X\hat{\beta}_{mle})$$

(b) Consider the null hypothesis for testing

$H_0 : \underline{\beta} = \underline{\beta}^0$ for the linear model

$$\underline{Y}^{n\times 1} = X^{n\times p}\beta^{p\times 1} + \underline{\in}^{n\times 1}$$

With rank (X) $= p$ and $v(\underline{Y}) = \sigma^2 I, \sigma^2$ being unknown

Let $\underline{Z} = \underline{Y} - X\beta^0$ and

$$Q_1 = \underline{Z}'X(X'X)^{-1}X'\underline{Z}$$

$$Q_2 = \underline{Z}'[1 - X(X'X)^{-1}X']\underline{Z}$$

Then show that

$\frac{Q_1}{\sigma^2}$ and $\frac{Q_2}{\sigma^2}$ are independently distributed. Further under $H_0$, show that both the quantities follow chi-square distributions with appropriate degrees of freedom. 4+6=10

2. (a) Derive the expected values of mean squares for two-way classified data with one observation per cell under the fixed and random models.

(b) In what respects do analysis of variance, regression analysis and analysis of covariance differ? 7+3=10

3. (a) Using the technique of analysis of variance how would you test the hypothesis that the regression equation is linear?

(b) Clearly bring out the differences among "fixed", "mixed" and "random" models. 7+3=10

4. (a) Consider the following random effects model $y_{ij} = \mu + b_i + \in_{ij}; i = 1....k, j = 1,2,...r$ with the following assumptions

$\in_{ij}$, $b_i$ are independently distributed. Find the estimates of $\sigma_b^2$ and $\sigma_\in^2$ .

(b) How would you modify the model and method if $b_i$ is assumed to be fixed? 6+4=10

P.T.O.

5. For two-way classified data with $m(\geq 1)$ observation per cell find the ANOVA table clearly mentioning the model, assumptions, hypothesis under fixed effects and random effects model. 10

6. Write short notes on the following :
   (a) Testing of general linear hypothesis.
   (b) Testing of equality of a number of regression equations. 5+5=10