B. Sc. (Honours) Semester-IV Examination 2017
Statistics (Honours)
Course: BSC-41
(Estimation)

Time : Three Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. (a) Define a sufficient statistic. State a theorem based on which you can obtain sufficient statistic. 2+2=4

   (b) Let T be a sufficient statistic for the parameter θ and let $\psi(T)$ be a one-to-one function of T. Show that $\psi(T)$ is also sufficient of θ. 3

   (c) Let $X_i (i = 1, 2, ..., n)$ be a random sample from a rectangular population $R(0, \theta)$. Obtain a sufficient statistic for θ.

2. (a) Let X_1, X_2 and X_3 be a random sample of size three from a Poisson population with parameter λ. Define the statistics $T_1 = X_1 + X_2 + X_3$ and $T_2 = X_1 + X_2 + 2X_3$. Show, from definition, that T_1 is sufficient λ and T_2 is not. 5

   (b) Let $X_i (i = 1, 2, ..., n)$ be a random sample of size n from a distribution with the p.d.f.

$$f_0(x) = \begin{cases} \dfrac{1}{\theta_2} \exp[-(x - \theta_1) / \theta_2] & \text{if } \theta_1 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

   Where $-\infty < \theta_1 < \infty, 0 < \theta_2 < \infty$. Derive sufficient statistics for the parameters θ_1 and θ_2. 5

3. (a) Discuss the criteria of a good estimator. In this regard define a minimum variance unbiased estimator of a parametric function $\gamma(\theta)$. 4

   (b) State and prove the Cramér-Rao Inequality for the variance of an estimator T of a parametric function $\gamma(\theta)$. 6

4. (a) Give the Cramér definition of efficiency $e(\theta)$ of an estimator. Let T_1 and T_2 be two unbiased estimators of the parametric function $\gamma(\theta)$ having the same variance. Show that the correlation coefficient (ρ_θ) between T_1 and T_2 cannot be smaller than $2e_\theta - 1$. 6

   (b) If X be a binomially distributed random variable with parameters n and p, show that given n, $1/p$ is not estimable. 4

5. (a) Suppose n objects are drawn at random, one at a time without replacement, from a lot of N objects of which M are of a particular kind. Let $X_1, X_2, \ldots, X_n$ are random variables, X_i assuming the value 1 or 0 according as the ith draw results or does not result in an object of the given kind. Suppose N is known and M is unknown. Derive a sufficient statistic for M. 5

   (b) Let U be an unbiased estimator of a parametric function $\gamma(\theta)$ and T be a sufficient statistic for θ. Discuss how one can get an estimator of $\gamma(\theta)$ having smaller variance than U. 5

P.T.O.

6. (a) Let T_1, T_2,…, T_k be independent unbiased estimators for the parameter θ with variances $\sigma_1^2, \sigma_2^2, ..., \sigma_k^2$ respectively. Find the Best Liner Unbiased Estimator of θ based on T_1, T_2,…,T_k. 5

(b) Define the likelihood function and discuss how one can use this function for estimating an unknown parameter θ. 5

7. (a) Given independent random variables X_1, X_2,…,X_n with the common density function

$$f_\theta(x) = K(\theta)\exp(-\sum_{i=0}^{m}\theta_i x^i)$$

Show that the method of moments agrees with method of maximum likelihood in estimating $\theta_i (i = 0,1,...,m)$. 5

(b) If T be a sufficient statistic for θ, then any solution of the likelihood equation will be a function of T. 3

(c) If T is the ML estimator of θ and $\psi(\theta)$ is a one-to-one function of θ, then $\psi(T)$ is the ML estimator for $\psi(\theta)$. 2