B.A. (Honours) Examination, 2023
Semester - I (NEP)
Subject: Economics
Course: MJEC02
(Mathematical Methods for Economics I)

Time: 3 Hours **Full Marks: 80**

Questions are of value as indicated in the margin
Answer any five (05) of the following questions

1. (a) Show that if f(x) is differentiable at $x = x_0$, then f(x) is also continuous at $x = x_0$.
(b) If a function is continuous over the closed interval [a,b] and differentiable over the open interval (a,b), then prove that there exists at least one point c∈(a,b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

7+9 = 16

2. (a) Prove that $\lim_{x\to 0} \frac{Sinx}{x} = 1$.
(b) Using the Cauchy's theorem on limit, prove that: $\lim_{n\to\infty}(\frac{1}{n^2} + \frac{1}{(n+1)^2} + \cdots + \frac{1}{(2n)^2}) = 0$.

8+8 = 16

3. (a) Find the 2nd degree Taylor Polynomial of $y = logx$ at $x = 1$.
(b) Using the first principle of differentiation, derive $\frac{dy}{dx}$ when $y = x^2 + 1$.

8+8 = 16

4. (a) Find the critical point(s) and the maximum or minimum value of the function $y = x^2 + 2x + 1$.
(b) A rectangular garden has to be built using a break wall as one side and wire fencing for the other three sides. Given 100 meters of wire fencing, determine the dimensions that creates a garden of maximum area. What is the maximum area?

6+10 = 10

5. (a) Graphically and mathematically interpret a convex combination of two points on a function.
(b) Differentiate the function $y = logx^2$ with respect to x, using first principle of differentiation.

7+9 = 16

6. (a) Derive $\frac{dy}{dx}$ from the equations: (i) $x^3 + y^3 + 3x^2y + 3xy^2 = 0$; (ii) $y = 5x^2 - e^y$
(b) Verify whether the functions satisfy the Rolle's theorem:
(i) $f(x) = x^2 + 2x$, over [-2,0]
(ii) $f(x) = 2x^2 - 8x + 6$, over [1, 3]

8+8 = 16

7. (a) Show that the statement P→Q is equivalent to the statement Q or Not P.
(b) What is wrong with the following argument?
Tom cats are cats.
Cats are species.
Tom cats are species.
(c) Which of the following statements are true (for all sets A, B and C)
(i) If A∈B and B ⊂ C then A ∈ C.
(ii) If A ∈ B and B ⊂ C then A ⊂ C.
(iii) Prove that A ∩ (B - C) ⊂ A – (B ∩ C)

4+4+4+4 = 16

8. (a) Briefly explain (with diagram) the concept of vector addition and scalar multiplication.

(b) Define dot product. Let $U = (1, -2, 3), V = (4,5, -1)$ *and* $W = (2,7,4)$ be the three vectors in R^3, then find which pair of vectors are orthogonal to each other.

(c) For a rectangular 2' × 3' × 4' box, find the angle that the longest diagonal makes with the 4' side.

(d) Define a unit vector. For a vector $U = (-1, 2, -3)$, find a vector of length $2/\sqrt{3}$ which points in the opposite direction.

4+4+4+4 = 16

9. (a) Define parametric representation of a line with diagram.

(b) Transform the following parameterized equation in to the form: $x_2 = mx_1+b$, for $x_1 = 3$, $x_2 = 5+t$.

(c) Transform $2x_2 = 3x_1 + 5$ into its parametric form.

(d) Draw a plane, and show the path you would traverse, were you to start at (-1, 3) and then displace yourself first by vector (1, -3) and then by vector (-1, -3).

(e) Write the equation of the plane through the point (3, 4, 5) with normal vector (6, 7, 8).

3+3+3+4+3 = 16

10. (a) State the conditions for a system of two non-degenerate linear equations in two unknowns have (i) one solution (ii) no solution (iii) infinite number of solutions with diagrams.

(b) L_1: $x - 3y - 2z = 6$; L_2: $2x - 4y - 3z = 8$; L_3: $-3x + 6y + 8z = -5$

For the above system, find the solution by Gaussian forward elimination and backward substitution method.

8+8 = 16

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