M.A. Examination, 2023
SEMESTER-I
Subject: ECONOMICS
Course: C-4
(Quantitative Economic Analysis)

Time: 3 Hours **Full marks: 40**

Questions are of value as indicated in the margin.

Answer **Question no 1** and **any three** from the rest

1. Draw the phase diagram and analyse the nature of stability of steady state growth equilibrium in the following two sector dynamic model, given by

   $\dot{k_1} = 12 - k_1 - k_2$; and

   $\dot{k_2} = \sqrt{k_1} - k_2$

   Here $\dot{k_i}$ is the capital labour ratio in the ith sector.

   10

2. a. Find the optimal control path that will

   Maximize $\int_0^1 (y - u^2)dt$

   Subject to $\dot{y} = u$ and $y(0) = 5$, $y(1)$ free.

   b. Derive using optimal control theory the shortest path between two points.

   5+5=10

3. a. Let $p(t)$ represent the consumer price index. If the rate of inflation of the price index is constant at 5%, and the price index has a base value of 100 at $t = 0$, solve the expression showing price index as a function of time.

   b. Suppose that a fish population grows according to the function

   $g(y) = 2y(1 - {}^{y}/_{2})$

   Where y is the stock of fish. The fish population is subjected to a constant level of harvesting by fishing industry. If the harvest is a constant equal to ${}^{3}/_{4}$, will the fish population reach a steady-state (positive) size?

   4+6=10

4 (a). Suppose 100 grams of apple contain 5 milligram (mg) of Vitamin-C, 110 mg of Potassium and 2 mg of Sodium. The same quantity of meat contains no Vitamin-C, 420 mg of Potassium and 60 mg of Sodium. It is prescribed that daily nutrient requirement of an adult is at least 90 mg of Vitamin-C and 3400 mg of Potassium. The daily Sodium intake, however, should not exceed 2300 mg. Prices of apple and meat are Rs. 200 and Rs. 700 per kg respectively. Assuming these two are the only sources of these nutrients, formulate an LPP that might be used to find out the least cost food mix (clearly specify your variables and explain the constraints).

(b) Find the dual of the problem you formulated in part (a) of this question and interpret the dual variables. 5+5=10

5. (a) Find the optimal solution of the following LPP:

$$\begin{aligned} \text{Maximize } & z = 4x_1 + 5x_2 \\ \text{Subject to, } & x_1 + 2x_2 \leq 10 \\ & 6x_1 + 6x_2 \leq 36 \\ & x_1 \leq 4 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(b) Consider the following LPP with two variables and four constraints, for which it is known that the optimal solution of this problem occurs at $x_1 = 8$ and $x_2 = 2/3$

$$\begin{aligned} \text{Maximize } & z = 3x_1 + 4x_2 \\ \text{Subject to, } & x_1 + x_2 \leq 10 \\ & 2x_1 + 3x_2 \leq 18 \\ & x_1 \leq 8 \\ & x_2 \leq 6 \\ \text{and } & x_1, x_2 \geq 0 \end{aligned}$$

Now, considering it as primal, construct its dual problem. There should be four dual variables as the primal has four constraints. Find the optimum values of those four dual variables, using the optimum values of the primal variables and with the help of the 'complementary slackness' property. Also calculate the values of the objective functions of primal and dual problems. 5+5=10

6 (a). The following table shows the inter-industry commodity flows in an economy with two sectors (I and II) within a year.

Industry	I	II	Final Consumption	Gross Output
I	500	1600	400	2500
II	1750	1600	4650	8000
Labour	250	4800		

(i) Construct the input-output coefficient matrix 'A'.

(ii) Calculate $[I - A]^{-1}$

(iii) Check the Hawkins-Simon condition.

(b) Using the I-O coefficients computed above, and assuming sufficient supply of labour, show the feasible set of gross output mix in the (X_1, X_2) plane where X_i denotes gross output of the i-th industry. (2+2+2) +4 = 10

7 (a). The technology matrix for a two-sector open I-O system is $\begin{bmatrix} 0.2 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}$ with labour input coefficients for the two sectors being 0.1 and 0.2 respectively. If the available labour in the economy is 800 units, find the equation of the consumption possibility frontier. Show it graphically.

(b). The production functions of a two-sector open input-output system are given below:

$X_1 = \text{Min}\left[\frac{X_{11}}{0.2}, \frac{X_{21}}{0.2}, \frac{X_{01}}{0.1}\right]$ $X_2 = \text{Min}\left[\frac{\cancel{X_{21}}^{\,12}}{0.7}, \frac{X_{22}}{0.2}, \frac{X_{02}}{0.2}\right]$

Where X_{ij}= The amount of i-th input used in j-th sector (1,2 are sectors and 0 represent labour)

Assuming perfectly competitive markets, find the equilibrium relative price ratio of the two commodities. 5+5=10