M.A. Examination, 2022
SEMESTER-I
Subject: ECONOMICS
Course: C-4
(Quantitative Economic Analysis)

Time: 3 Hours **Full marks: 40**

Questions are of value as indicated in the margin.
Answer **Question no 1** and **any three** from the rest

1. Find the optimal control path for the following problems that will

(a) Maximize $\int_0^T -(au + bu^2)dt$, subject to $\dot{y} = y - u$ and $y(0) = y_0$, $y(t)$ free.

(b) Maximize $\int_0^2 (2y - 3u)dt$, subject to $\dot{y} = y + u$, $y(0) = 4$, $y(2)$ free and $u(t) \epsilon [0,2]$ 4+6

2 (a) Solve the following equations and ensure that the initial conditions are satisfied

$$\ddot{y} + \dot{y} + \frac{1}{4}y = 2 \text{ with } y_0 = 10 \text{ and } \dot{y}_0 = 8.$$

(b) Increase in carbon dioxide in the earth's atmosphere has been cited as a probable cause of global warming. Let y represent the stock of cabon dioxide and x (> 0, a constant) represent the flow of carbon dioxide emissions that come from industrial activities. Assume that the dynamics of y is given by $\dot{y} = x - y^a$

where the term y^a represents earth's capacity to remove the carbon dioxide from atmosphere and allow its absorption elsewhere (i.e., in trees, oceans). Conduct a qualitative analysis of this model (i) for the case where $a > 0$ and (ii) for the case where $a < 0$. Comment on your results. 5+5

3. Find the complete solution to the following system of differential equations with $y_1(0) = 1$ and $y_2(0) = 3$:

$$\dot{y}_1 = y_1 - 3y_2 - 5$$
$$\dot{y}_2 = \frac{1}{4}y_1 + 3y_2 - 5$$

10

4. (a) A furniture trader sells two items - tables and chairs. He does not produce anything and sells all the items that are bought from other manufacturers. He can invest Rs.10000 and a space that can store at most 60 pieces of furniture. A table cost him Rs.500 and a chair Rs.200 to buy from the manufacturers. He can make a profit of Rs. 50 per table and Rs. 30 pêr chair that he sells. Formulate his optimization problem as an LPP, clearly defining all the variables you used.

(b) Solve the following LPP where x_1 and x_2 are the production quantities of two commodities and the right hand side of the three constraints indicate the availability of three resources respectively.

$$\begin{aligned} \textit{Maximize} \quad & z = 9x_1 + 8x_2 \\ \textit{Subject to,} \quad & 4x_1 + 3x_2 \leq 360 \\ & 2x_1 + 3x_2 \leq 180 \\ & 2x_1 + x_2 \leq 100 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(c) From the solution of the problem above (part b), can you identify a resource which does not have a positive valuation to the producer? Explain your answer. **4+4+2=10**

5. Suppose two commodities are being produced with three inputs. The producer's revenue maximization problem is specified as follows.

$$\begin{aligned} \textit{Maximize} \quad & z = p_1x_1 + p_2x_2 \\ \textit{Subject to,} \quad & a_{11}x_1 + a_{12}x_2 \leq b_1 \\ & a_{21}x_1 + a_{22}x_2 \leq b_2 \\ & a_{31}x_1 + a_{32}x_2 \leq b_3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Where p_i and x_i are prices (rupees) and quantities (units) of the two commodities (i=1,2) ; b_j is the quantity available of the j-th input (j=1,2,3) and a_{ij} are the input coefficients following fixed coefficient technology.

(a) Construct the dual problem and interpret the dual variables.

(b) Rewrite the primal and dual constraints by introducing appropriate slack and surplus variables and find the relationship between primal/dual variables and slack/surplus variables at their optimum values. **3+7=10**

6. The coefficient matrix for a two-sector open I-O system is given as $\begin{bmatrix} 0.2 & 0.2 \\ 0.7 & 0.2 \end{bmatrix}$.

(a) Find out the optimum gross production levels of the two commodities if the final consumptions of the two sectors are 200 and 400 units respectively.

(b) Assuming X_1 and X_2 are the gross production quantities of the two sectors, formulate the two inequality constraints involving X_1 and X_2 following the coefficient matrix given above. Show the feasible set of production mix in the (X_1, X_2) plane using a diagram. **5+5=10**

7. The coefficient matrix for a two-sector open I-O system is $\begin{bmatrix} 0.2 & 0.2 \\ 0.7 & 0.2 \end{bmatrix}$ and the labour coefficients for the two sectors are 0.1 and 0.6 respectively. Total available labour for the economy is 50 units and the wage rate of labour is Rs 30 per unit.

(a) Assuming C_1 and C_2 are the final consumption quantities, find the equation of the Consumption Possibility Frontier.

(b) Find the absolute prices of the two commodities. Clearly describe your assumptions regarding your formulation of the price equations. **4+6=10**