B.A. (Honours) Examination, 2022
Semester - II (CBCS)
Subject: Economics
Course: CC-04 (Mathematical Methods in Economics - II)

Time: 3 Hours **Full Marks: 60**

Questions are of value as indicated in the margin

Answer any four (04) of the following questions

1. (i) Find a 2×2 orthogonal matrix P whose first row is a multiple of (3,-4).

(ii) Find an upper triangular matrix such that $A^3 = \begin{bmatrix} 8 & -76 \\ 0 & 27 \end{bmatrix}$.

(iii) Let $A = \begin{bmatrix} 5 & 2 \\ 0 & k \end{bmatrix}$, then find all numbers k may take for which A is a root of the polynomial $f(x) = x^2 - 7x + 10$.

(iv) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$, Find a 2×3 matrix B with distinct non zero entries such that AB = 0. 4+3+4+4 = 15

2. (i) Show that $A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$ can be expressed as a sum of a symmetric and a skew-symmetric matrix.

(ii) Compute the Adjoint of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 1 & -1 \end{bmatrix}$ and the inverse of the matrix (A^{-1}).

(iii) Solve the given set of equations by matrix method: x-3y = 4, 3x-y = 5. 4+8+3 = 15

3. (i) Prove that X-1 is a factor of the following determinant: $\begin{vmatrix} X+1 & 3 & 5 \\ 2 & X+2 & 5 \\ 2 & 3 & X+4 \end{vmatrix}$

(ii) Prove without expanding that $\begin{vmatrix} bc & a^2 & a^2 \\ b^2 & ca & b^2 \\ c^2 & c^2 & ab \end{vmatrix} = \begin{vmatrix} bc & ab & ca \\ ab & ca & bc \\ ca & bc & ab \end{vmatrix}$.

(iii) Solve by Cramer's Rule: 2x-z = 1, 2x+4y-z = 1, x-8y-3z = -2 5+5+5=15

4. Define the Local and Global maxima in case of a function of several variables. Find the critical point(s) of the following function, and use the second partials test to find any local extrema or saddle point:

$$f(x,y) = 4x^2 + 9y^2 + 8x - 36y + 20$$ 3+12 = 15

5. Find the linear approximation L(x,y) and the quadratic approximation Q(x,y) of the following function near the point (0, 0): $f(x,y) = Sin2x + cosy$. 7+8 = 15

6. Let $f(x) = -x^2 + 2ax + 4a^2$ be a one variable function where 'a' is a parameter. Derive the Optimum value function $f^*(a)$ and then verify the envelop theorem. 5+10 = 15

7. Find the optimum values of the following functions using Lagrange multiplier and verify whether the optimum values are maximum or minimum:

Objective function: $f(x,y) = x^2 + y^2$; Constraint: $xy = 1$

Objective function: $f(x,y) = (6 - x^2 - y^2)^{1/2}$; Constraint: $x + y = 2$ 7+8 = 15

8. Suppose a firm maximizes its output $Q = L^\alpha K^{1-\alpha}$ subject to its cost $C = wL + rK$, where all symbols follow their usual meanings. Proof the Euler's theorem. If there is another problem where the firm minimizes its cost subject to a given level of output, then show that both problems provide same outcome in the process of optimization. 7+8 = 15

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