B.A. (Honours) Examination, 2023
Semester - I (CBCS)
Subject: Economics
Course: CC-02
(Mathematical Methods for Economics - I)

Time: 3 Hours

Full Marks: 60

Questions are of value as indicated in the margin

Answer any four (04) of the following questions

1. (a) Show that if f(x) is differentiable at $x = x_0$, then f(x) is also continuous at $x = x_0$.
(b) If a function is continuous over the closed interval [a,b] and differentiable over the open interval (a,b), then prove that there exists at least one point c∈(a,b) such that $f'(c) = \frac{f(b) - f(a)}{b-a}$.

7+8 = 15

2. (a) Prove that $\lim_{x\to 0} \frac{Sinx}{x} = \cancel{0}.\ 1.$
(b) Using the Cauchy's theorem on limit, prove that: $\lim_{n\to\infty}(\frac{1}{n^2} + \frac{1}{(n+1)^2} + \cdots + \frac{1}{(2n)^2}) = 0.$

7+8 = 15

3. (a) Find the 2nd degree Taylor Polynomial of $y = logx$ at $x = 1$.
(b) Using the first principle of differentiation, derive $\frac{dy}{dx}$ when $y = x^2 + 1$.

8+7 = 15

4. (a) Find the critical point(s) and the maximum or minimum value of the function $y = x^2 + 2x + 1$.
(b) A rectangular garden has to be built using a break wall as one side and wire fencing for the other three sides. Given 100 meters of wire fencing, determine the dimensions that creates a garden of maximum area. What is the maximum area?

6+9 = 15

5. (a) Graphically and mathematically interpret a convex combination of two points on a function.
(b) Differentiate the function $y = logx^2$ with respect to x, using first principle of differentiation.

7+8 = 15

6. (a) Derive $\frac{dy}{dx}$ from the equations: (i) $x^3 + y^3 + 3x^2y + 3xy^2 = 0$; (ii) $y = 5x^2 - e^y$
(b) Verify whether the functions satisfy the Rolle's theorem:
(i) $f(x) = x^2 + 2x$, over [-2,0]
(ii) $f(x) = 2x^2 - 8x + 6$, over [1, 3]

7+8= 15

7. (a) Briefly explain (with diagram) the concept of vector addition and scalar multiplication.
(b) For a rectangular 2' × 3' × 4' box, find the angle that the longest diagonal makes with the 4' side.
(c) Define a unit vector. For a vector $U = (-1, 2, -3)$, find a vector of length $2/\sqrt{3}$ which points in the opposite direction.

5+5+5 = 15

8. (a) State the conditions for a system of two non-degenerate linear equations in two unknowns have (i) one solution (ii) no solution (iii) infinite number of solutions with diagrams.
(b) L_1: $x - 3y - 2z = 6$; L_2: $2x - 4y - 3z = 8$; L_3: $-3x + 6y + 8z = -5$
For the above system, find the solution by Gaussian forward elimination and backward substitution method.

7.5+7.5 = 15

****