B. Sc. (Honours) Semester-II Examination 2017
Statistics (Honours)
Course: BSC-21 (New & Old)
(Probability Distribution II)

Time : Three Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. (a) If the probability of success in a Bernoullian experiment is .01, how many trials are necessary in order to achieving probability of at least one success as ½ ?
   (b) Let X be the number of heads in n throws of a biased coin with probability of getting head as p. Establish mathematically that X and n-X are negatively correlated.
   (c) In a sequence of Bernoullian trials with probability p of success, find the probability that a successes will occur before b failures. 2+3+5=10
2. (a) Suppose that the number of eggs laid by an insect following a Poisson distribution with parameter λ and the probability of an egg developing is p. Assuming mutual independence of the events, show that the number of surviving insects follows a Poisson distribution with parameter λp.
   (b) Let X_1, X_2 be independent random variable each having geometric distribution with mass function $f_x(x) = pq^x, X = 0,1,2,...$. Show that the conditional distribution of X_1 given $X_1 + X_2$ is rectangular. 6+4=10
3. (a) Prove that for a normal distribution with mean μ and variance σ^2, mean deviation about mean is .7979σ.
   (b) Show that for $x > 0$

$$1-\Phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \left\{ \frac{1}{x} - \frac{1}{x^3} + \frac{1.3}{x^5} - \frac{1.3.5}{x^7} + \cdots \right\}$$

   Hence $1-\Phi(x) < \phi(x)$

   where $\phi(x)$ is the distribution function of standard normal variable. $\phi(x)$ is the ordinate at the point x. 4+6=10
4. (a) Derive the r th order raw moment of a lognormal distribution. Hence find the mean and variance of the distribution.
   (b) Find the first and third quartiles of a Cauchy distribution with parameter λ and μ. Hence find its quartile deviation. 6+4=10
5. (a) Define a bivariate normal distribution clearly mentioning the parameters. Obtain its moment generating function.
   (b) The random variables X and Y have the circular uniform distribution given by the p.d.f.

$$f(x,y) = \begin{cases} \frac{1}{\pi r^2} & for \quad x^2 + y^2 < r^2 \\ 0 & otherwise \end{cases}$$

   What are the marginal and conditional distributions? 6+4=10

P.T.O.

6 (a) How many times an unbiased coin must be tossed in order that the probability will be at least .90 that is the proportion of the number of heads will be between .4 and .6?

(b) The discrete random variable X has the power series distribution with p.m.f.

$$f_x(x) = a_x \frac{\theta^x}{g(\theta)} \text{ for } X = 0,1,2,...$$

Where $g(\theta)$ is a differentiable function of the parameter θ.

Show that $E(X) = \theta \frac{d}{d\theta}[\log_e g(\theta)]$ and

$$V(X) = E(X) + \frac{\theta^2 d^2}{d\theta^2}[\log_e g(\theta)]$$

4+6=10

7. (a) Let $\{F_n\}$ be a sequence of distribution functions such that

$$F_n(x) = \begin{cases} 0 \quad , \quad x < \theta + \frac{1}{n} \\ 1 \quad , \quad x \geq \theta + \frac{1}{n} \end{cases}$$

show that the corresponding sequence of random variables $X_n \xrightarrow{L} X$ where X is a random variable degenerate at X = θ.

(b) Write the statement of weak law of large number. Determine if the following obeys this law or not

$$P\{X_k = 2^k\} = P\{X_k = -2^k\} = \frac{1}{2^{2k+1}}$$

$$P\{X_k = 0\} = 1 - \frac{1}{2^{2k}}.$$

5+5=10
